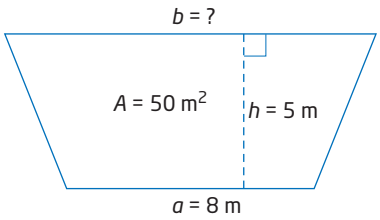
4.3 Solve Equations Involving Fractions

Rock gardens are a fascinating tradition of the Japanese culture. Their beauty is a result of a careful combination of natural landscape and artistic design. The peaceful atmosphere of a Japanese rock garden can provide moments of quiet reflection, contemplation, and appreciation of the simple beauties in life. How can mathematics be used in the design of a rock garden?

Investigate

What techniques can you use to solve equations involving fractions?

Kumiko is designing a rock garden in the shape of a trapezoid. She decides that the garden should have a front width of 8 m and a depth of 5 m. The area must be 50 m^2 to fit her design. How wide should Kumiko's garden be at the back?



1. The formula for the area of a trapezoid is $A = \frac{(a + b)h}{2}$. Substitute the known values into this formula.
2. Find the value of the unknown, b.
3. **Reflect** Describe any difficulties you encountered in solving this equation. What can you do to make it easier to solve equations involving fractions?

Example 1 Solve Equations Involving One Fraction

Solve.

a) $6 = \frac{1}{3}(8 + x)$ **b)** $\frac{3(y - 5)}{4} = 7$

Solution

When solving an equation involving fractions, it is helpful to multiply both sides by the same value to eliminate the fractions.

a)

$$6 = \frac{1}{3}(8 + x)$$

Instead of distributing a fraction to remove brackets, multiply both sides of the equation by 3 to eliminate the fraction on the right side.

$$3 \times 6 = 3 \times \frac{1}{3}(8 + x)$$

$$18 = 8 + x$$

Subtract 8 from both sides.

$$18 - 8 = 8 + x - 8$$

$$10 = x$$

b) Method 1: Pencil and Paper

$$\frac{3(y - 5)}{4} = 7$$

Multiply both sides of the equation by 4 to eliminate the fraction.

$$4 \times \frac{3(y - 5)}{4} = 4 \times 7$$

Apply the distributive property to remove brackets.

$$3(y - 5) = 28$$

$$3y - 15 = 28$$

Add 15 to both sides.

$$3y - 15 + 15 = 28 + 15$$

$$3y = 43$$

Divide both sides by 3.

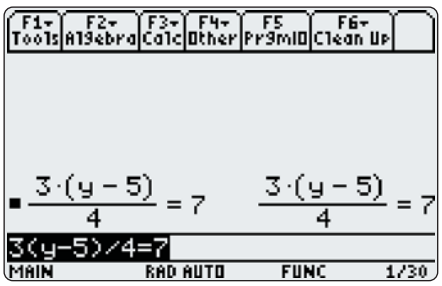
$$\frac{3y}{3} = \frac{43}{3}$$

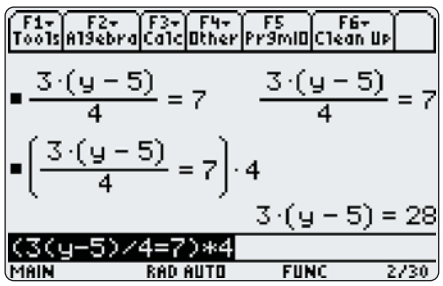
$$y = \frac{43}{3}$$

Method 2: Computer Algebra System (CAS)

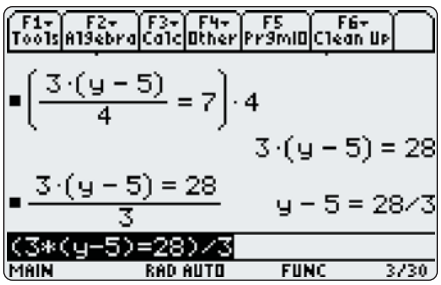
In the Home screen, enter the equation $\frac{3(y - 5)}{4} = 7$.

Multiply by 4 to eliminate the fraction.



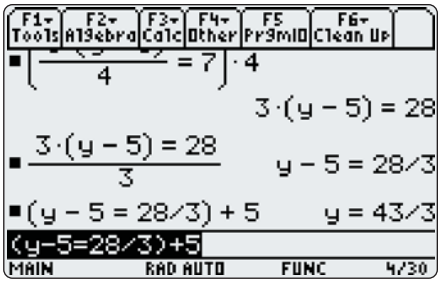


You can distribute the 3, or you can simply divide both sides by 3 to remove the brackets.



This produces an unpleasant fraction on the right side. However, a CAS has no difficulty working with fractions. Finish the solution by adding 5 to both sides.

The solution is $y = \frac{43}{3}$.



Example 2 Solve Equations With More Than One Fraction

a) Solve $\frac{k+2}{3} = \frac{k-4}{5}$.

b) Solve and check $\frac{1}{3}(2x-5) = \frac{3}{4}(x-2)$.

To find the LCD for $\frac{1}{3}$ and $\frac{1}{5}$, I'll count by 3s and 5s.
3 6 9 12 (15)
5 10 (15)
The LCD is 15.

Solution

a)

$$\frac{k+2}{3} = \frac{k-4}{5}$$

$$\overset{5}{\cancel{15}} \times \frac{k+2}{\cancel{3}_{1}} = \overset{3}{\cancel{15}} \times \frac{k-4}{\cancel{5}_{1}}$$

Multiply both sides by the lowest common denominator to eliminate both fractions.

$$5(k+2) = 3(k-4)$$

Apply the distributive property to remove brackets.

$$5k + 10 = 3k - 12$$

$$5k + 10 - 3k - 10 = 3k - 12 - 3k - 10$$

Subtract $3k$ and 10 from both sides.

$$2k = -22$$

$$\frac{2k}{2} = \frac{-22}{2}$$

Divide both sides by 2.

$$k = -11$$

b) $\frac{1}{3}(2x - 5) = \frac{3}{4}(x - 2)$

I'll use prime factors to find the LCD.
$3 = 3 \quad 4 = 2 \times 2$
The LCD is $3 \times 2 \times 2$, or 12.

$12 \times \frac{1}{3}(2x - 5) = 12 \times \frac{3}{4}(x - 2)$ — Multiply both sides by the lowest common denominator to eliminate both fractions.

$\frac{\overset{4}{\cancel{12}}}{1} \times \frac{1}{\underset{1}{\cancel{3}}}(2x - 5) = \frac{\overset{3}{\cancel{12}}}{1} \times \frac{3}{\underset{1}{\cancel{4}}}(x - 2)$

$4(2x - 5) = 9(x - 2)$ — Apply the distributive property to remove brackets.

$8x - 20 = 9x - 18$

$8x - 20 - 8x + 18 = 9x - 18 - 8x + 18$ — Subtract $8x$ from both sides and add 18 to both sides.

$-2 = x$

Check:

L.S. $= \frac{1}{3}(2x - 5)$
$= \frac{1}{3}[2(-2) - 5]$
$= \frac{1}{3}(-4 - 5)$
$= \frac{1}{3}(-9)$
$= -3$

R.S. $= \frac{3}{4}(x - 2)$
$= \frac{3}{4}[(-2) - 2]$
$= \frac{3}{4}(-4)$
$= -3$

Substitute $x = -2$ into the left side and right side of the original equation.

L.S. = R.S.

Since L.S. = R.S., $x = -2$ is the root of this equation.

Key Concepts

- You can simplify equations involving one fraction by multiplying both sides by the denominator of the fraction.
- When eliminating more than one fraction, find the lowest common denominator and multiply all terms on both sides of the equation by this value.

Communicate Your Understanding

C1 Explain the first step you would take to solve each equation.

a) $\frac{1}{4}(x - 3) = 5$ **b)** $5 - k = \frac{2}{3}$

C2 Without solving, identify which of these equations has the solution $k = 2$.

A $\frac{2}{3}(5 - k) = 2$ **B** $k - 1 = \frac{3}{4}$ **C** $-3 = \frac{k - 17}{5}$

C3 By what value should you multiply both sides of each equation to eliminate all fractions? Explain.

a) $\frac{x + 1}{5} = \frac{x}{6}$ **b)** $\frac{y + 2}{9} = \frac{y - 3}{3}$

Practise

For help with questions 1 and 2, see Example 1.

1. Solve using pencil and paper.

a) $\frac{1}{3}(x - 2) = 5$

b) $4 = -\frac{2}{3}(p - 2)$

c) $\frac{m + 4}{3} = 7$

d) $-14 = \frac{2(h - 3)}{5}$

2. Solve using a CAS. Use at least two steps.

a) $\frac{y - 4}{5} = -6$

b) $\frac{1}{4}(u - 5) = -2$

c) $3 = \frac{2}{5}(n + 7)$

d) $16 = \frac{3(v + 7)}{2}$

For help with questions 3 and 4, see Example 2.

3. Find the root of each equation. Check your answers.

a) $\frac{m - 3}{4} = \frac{m + 1}{3}$

b) $\frac{w - 1}{4} = \frac{w + 2}{3}$

c) $\frac{1}{4}(x - 3) = \frac{1}{3}(x - 2)$

d) $\frac{1}{5}(y - 3) = \frac{1}{6}(y + 4)$

4. Find the root of each equation. Use a CAS to check your answers.

a) $\frac{2}{3}(5n - 1) = -\frac{3}{5}(n + 2)$

b) $-\frac{3}{4}(d + 3) = \frac{4}{5}(3d - 2)$

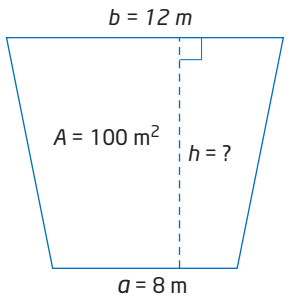
c) $\frac{3c - 2}{5} = \frac{2c - 1}{3}$

d) $\frac{5 - 2a}{4} = \frac{6 - a}{5}$

Connect and Apply

5. A trapezoidal backyard has an area of 100 m^2. The front and back widths are 8 m and 12 m, as shown.

What is the length of the yard from front to back?



6. Each of the following solutions contains an error. Identify the error and describe how to correct it.

a)

$$\frac{x-3}{5} = \frac{x+1}{4}$$
$$5(x-3) = 4(x+1)$$
$$5x - 15 = 4x + 4$$
$$5x - 15 - 4x + 15 = 4x + 4 - 4x + 15$$
$$x = 19$$

b)

$$\frac{1}{3}(3y-2) = \frac{1}{4}(y+3)$$
$$12 \times \frac{1}{3}(3y-2) = 12 \times \frac{1}{4}(y+3)$$
$$3y - 2 = y + 3$$
$$3y - 2 - y + 2 = y + 3 - y + 2$$
$$2y = 5$$
$$\frac{2y}{2} = \frac{5}{2}$$
$$y = \frac{5}{2}$$

7. The equation $C = \frac{5}{9}(F - 32)$ allows you to convert between Celsius and Fahrenheit temperatures. C is the temperature in degrees Celsius (°C) and F is the temperature in degrees Fahrenheit (°F).

a) A U.S. weather station predicts that the overnight low temperature will reach 25°F. What is this in degrees Celsius?

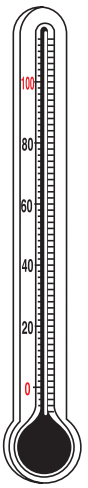
b) Room temperature is approximately 20°C. What is this equivalent to in degrees Fahrenheit?

8. Find the height of a triangle with base 10 cm and area 50 cm^2.

9. A garden is in the shape of a right triangle. The base of the triangle is 12 m, and the garden covers an area of 30 m^2. What length of fence is needed to surround the garden?

Did You Know?

The Celsius scale is based on two key properties of water. Under normal conditions, water freezes at 0°C and boils at 100°C. Splitting the temperature difference between these two points into 100 equal intervals produces the Celsius scale.



Achievement Check

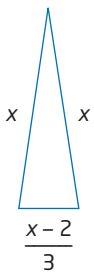
10. A backyard has a perimeter of 144 m.

a) If the backyard is square, what are the dimensions?

b) If the backyard is rectangular, and the length is three times the width, what are the dimensions?

c) If the backyard is a triangle, as shown, write an algebraic expression for the perimeter and find its dimensions.

d) Which of the three backyard designs has the greatest area? Which has the least?



Extend

11. Solve.

a) $\frac{3p}{4} + \frac{p-5}{3} = \frac{1}{2}$

b) $\frac{u-3}{4} - 2 = \frac{3u}{2} + \frac{2u+1}{5}$

12. Fahad called Sara over. "I'm stuck on this question about equations," he said.

"Where did you get it from?" asked Sara.

"From my older brother's book," Fahad told her. "It asks you to find the roots of $x(x-5) = 0$. I know how to solve equations, but this one is different."

Sara said "I know! The answer is $x = 0$! If I substitute 0, it works!"

"Great!" said Fahad. "But the book says there are two roots."

"Well," said Sara, "if $a \times b = 0$, then either $a = 0$ or $b = 0$."

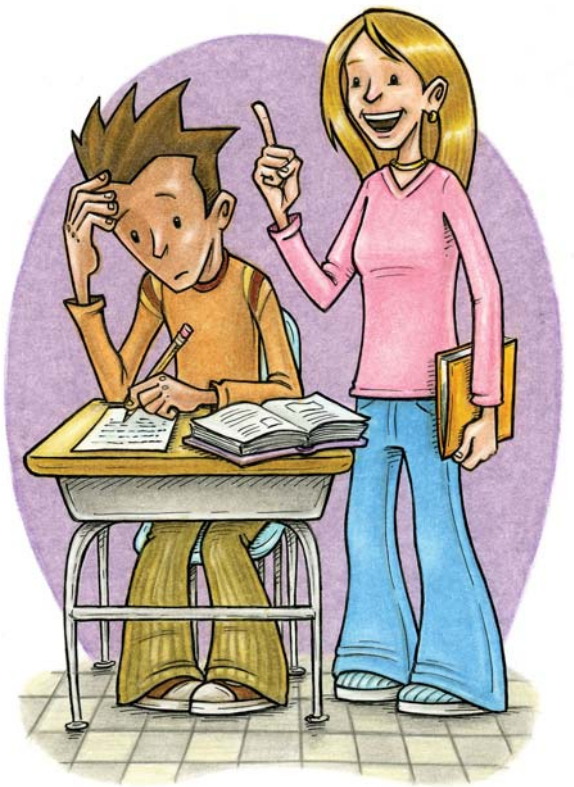
"I've got it!" said Fahad. "The other solution is $x = 5$, because it makes the value of the second bracket zero."

"Excellent!" said Sara.

Solve these questions from Fahad's brother's book.

a) $(x-3)(x-7) = 0$ (2 roots)

b) $x(x-4)(x+2) = 0$ (3 roots)



13. **Math Contest** Diophantus of Alexandria was born around the year 200. He is known as the Father of Arithmetic. A puzzle about Diophantus is as follows:

"His boyhood lasted one sixth of his life. He married after one seventh more. His beard grew after one twelfth more and his son was born 5 years later. The son lived to half his father's final age, and the father died 4 years after the son."

How old was Diophantus when he died?